MATH1520AB 2021-22 Quiz 1 (week 2) Solution

Full marks: 10 marks

Time allowed: 15 minutes

1. Let $X = \{1, 2, 3, 5, 8, 13\}$ and $Y = (2, 10]$. Find

   (a) $X \cap Y$ (1 mark)
   (b) $X \setminus Y$ (1 mark)

   Answer.

   (a) $X \cap Y = \{3, 5, 8\}$
   (b) $X \setminus Y = \{1, 2, 13\}$

2. Determine the natural domain of the following functions.

   (a) $f(x) = \dfrac{x^2 - 4}{x + 2} + \sqrt{x - 2}$ (2 marks)

   (b) $f(x) = \dfrac{1}{\sqrt{x^2 + 2x - 15}}$ (2 marks)

   Answer.

   (a) $D_f = \{x \in \mathbb{R} \mid x \neq -2 \text{ and } x - 2 \geq 0\} = [2, \infty)$

   (b) $f(x) = \dfrac{1}{\sqrt{x^2 + 2x - 15}} = \dfrac{1}{\sqrt{(x - 3)(x + 5)}}$

   $D_f = \{x \in \mathbb{R} \mid (x - 3)(x + 5) > 0\} = \{x \in \mathbb{R} \mid x < -5 \text{ or } x > 3\} = (-\infty, -5) \cup (3, \infty)$

3. Write $f(x) = |4x - 1| + |x + 3|$ as a piecewise function.   (2 marks)
   Answer.

   $$f(x) = \begin{cases} -(4x - 1) - (x + 3) = -5x - 2, & \text{if } x < -3 \\ -(4x - 1) + (x + 3) = -3x + 4, & \text{if } -3 \leq x < \frac{1}{4} \\ (4x - 1) + (x + 3) = 5x + 2, & \text{if } x \geq \frac{1}{4} \end{cases}$$

4. Let $f(x) = 2x^2 - x + 5$ and $g(x) = -3x + 2$. Find

   (a) $f(g(x))$ (1 mark)
   (b) $g(f(x))$ (1 mark)

   Answer.

   (a) $f(g(x)) = 2(-3x + 2)^2 - (-3x + 2) + 5 = 18x^2 - 21x + 11$
   (b) $g(f(x)) = -3(2x^2 - x + 5) + 2 = -6x^2 + 3x - 13$